Trippy Trojans

Graham Breed 11 July MMXI

> Take but degree away, untune that string,
> And, hark, what discord follows!
>
> *Ulysses, Troilus and Cressida I:III (Shakespeare)*

Motivation

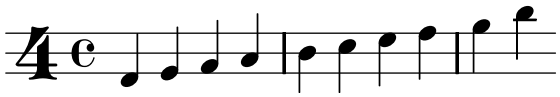
TRIPOD NOTATION is a way of writing music for Marvel temperaments, in particular Magic.[1] It's a useful composer's notation because it's simple and it makes the structure of the music clear. It is, however, not something we should expect performers – or anybody other than me – to be able to read.

Trojan notations are part of the Sagittal project.[2]. The use the common 12 equally tempered pitches along with new accidentals that equally divide the equally tempered semitones. The most generally useful Trojan division is 72 note equal temperament, where the semitone is divided into 6 twelfth-tones or *sextulas*.[3] It implies Marvel tempering and so is compatible with Tripod notation. The Trojan division that fits Magic temperament is 60 note equal temperament (tenth-tones or *quintulas*[4]).

In this PDF, then, you'll find examples of Tripod notation with equivalents in Trojan notations for 60 and 72 divisions of the octave. I made all of them with LilyPond.[5] LilyPond makes it convenient to render the same piece in different notations. This PDF is also documentation of how that works.

Implementation

BOTH TRIPOD AND TROJAN notations are implemented using include files. A simple example is the tripod scale:



The code to produce it is:

```
\include "tripod60.ly"
\new Staff {
    \fourclef \tripod #4 {
        yan tan eth meth pip
        seth leth hov dov yan'
    }
}
```

The include file `tripod60.ly` covers all the logic for handling Tripod notation, and sets the tuning to 60 note equal temperament. It's included, along with any other files you need to duplicate these examples, in `http://x31eq.com/magic/tripod-code.zip`.

The `\fourclef` command specifies the Tripod clef you want to use. It needs to be there because it helps to setup the right number of lines. Then, `\tripod #4` transposes the music so that it's in the correct register. You can use `\transpose yan yan'` if you prefer.

The pitches are specified using the short *yan tan tethera* names from *Tripod Notation*. This is what makes the system *trippy*. It takes a bit of getting used to the new names, but it's the simplest way because LilyPond reserves numerals for durations. Terse names would work as long as you don't mind some of them

[1] Introduced in *Tripod Notation*, `http://x31eq.com/magic/tripod.pdf`.

[2] `http://sagittal.org/`

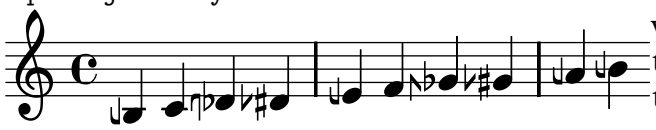
[3] From the Latin *sextula* for a sixth part of an *uncia* (one twelfth). (Jan Gullberg, *Mathematics: Frome the Birth of Numbers*, p. 16, Norton 1996.) I added an "s" instead of guessing the correct Latin plural.

[4] Gullberg doesn't list this, but it sounds right.

[5] See `http://x31eq/lilypond/` for instructions on microtonal use of LilyPond.

looking like diatonic names.

To get Trojan instead of Tripod notation, you can replace `tripod60.ly` with `trip-trojan60.ly`. The result looks like this:



By default, the `\fourclef` maps to treble. Sometimes you will need to specify the Trojan clef independent of the Tripod one. You can do that using variables. If you want to break voices at different parts for different notations, the code might get complicated. Unfortunately there's no way of avoiding the inherent complexity of the problem you need to solve.

A Red, Red Rose

I USED THIS SONG as an example in *Tripod Notation*. You can see it notated, including the unprepared dissonance that made sense at the time, in different ways in Figure 1. Because it started out diatonic, it returns to its nature in the Trojan notations. The chords it uses work fine in dydimic tuning (or 5-limit just intonation as the standard jargon would have it).

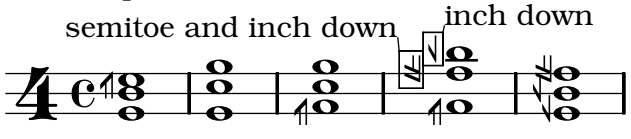
The third degree of each major chord is lowered by a didymic comma, and the root and fifth of each minor chord are lowered. That helps the tuning conform to the theoretical ideal behind Tripod notation. It also helps you to distinguish major and minor triads: count the distinct commas. Maybe you're so familiar with staff notation that you instinctively know where the major and minor chords should be. Think back to before you achieved this enlightenment, and appreciate how standard notation fails to make the distinction clear.

Seeing the music in a different notation can help you spot mistakes. This is a good example: originally, I had the second to last note of the melody as `meb` instead of `mef` so that it was out of tune with the accompaniment. That's a mistake that's been in the code for a few years. You can spot it in *Tripod Notation*. I only found it when I was thinking about how to explain the differences between the 60 and 72 note versions, and tried to work out why there were two different tunings of E.

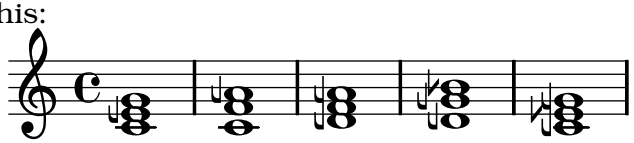
So, what are the differences between this song quantized to either 60 or 72 pitches to the octave? The two renderings are close enough that you could ignore their equal tempered origins and think of the comma shifts relative to a chain of pure fifths. The only difference is that the D of "that's" has a double sextula shift in 72. Maybe that's a mistake. It probably works either way. Oh, and the accidental symbols are a different shape. That's the other thing.

Didymic Comma Pump

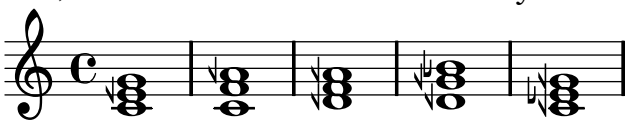
THE COMMA SHIFTS do a bit more work in a Didymic comma pump. Here's the original, in Tripod notation:



In 60 note equal temperament, it looks like this:



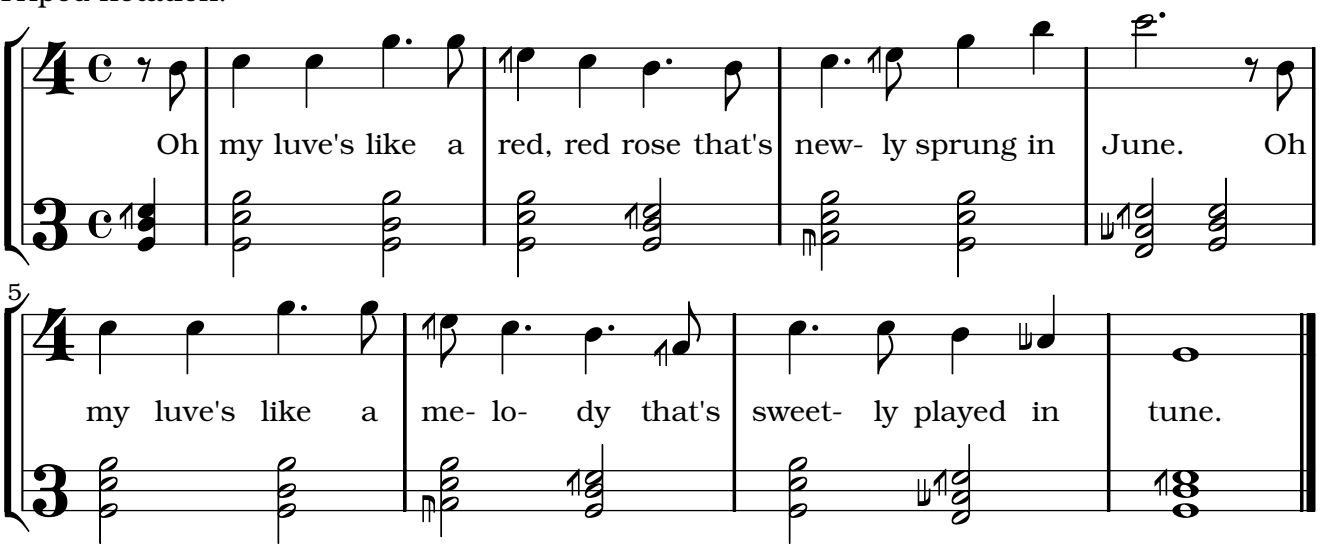
You can see that the last chord is like the first, but a quintula flatter. In 72 note equal temperament it's exactly the same, but with different accidental symbols:



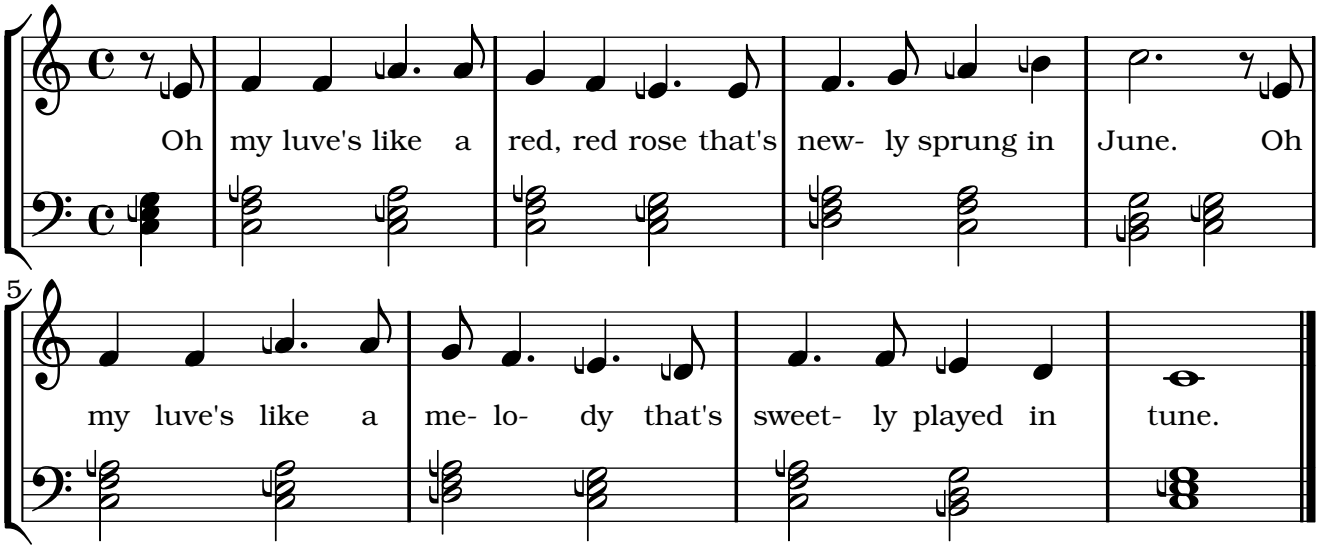
This kind of thing would happen all the time if conventional music were tuned Didymically. But, because it isn't, the notation can sweep the theoretical discrepancies under the metaphorical rug.

Figure 1: *A Red, Red Rose* in different notations.

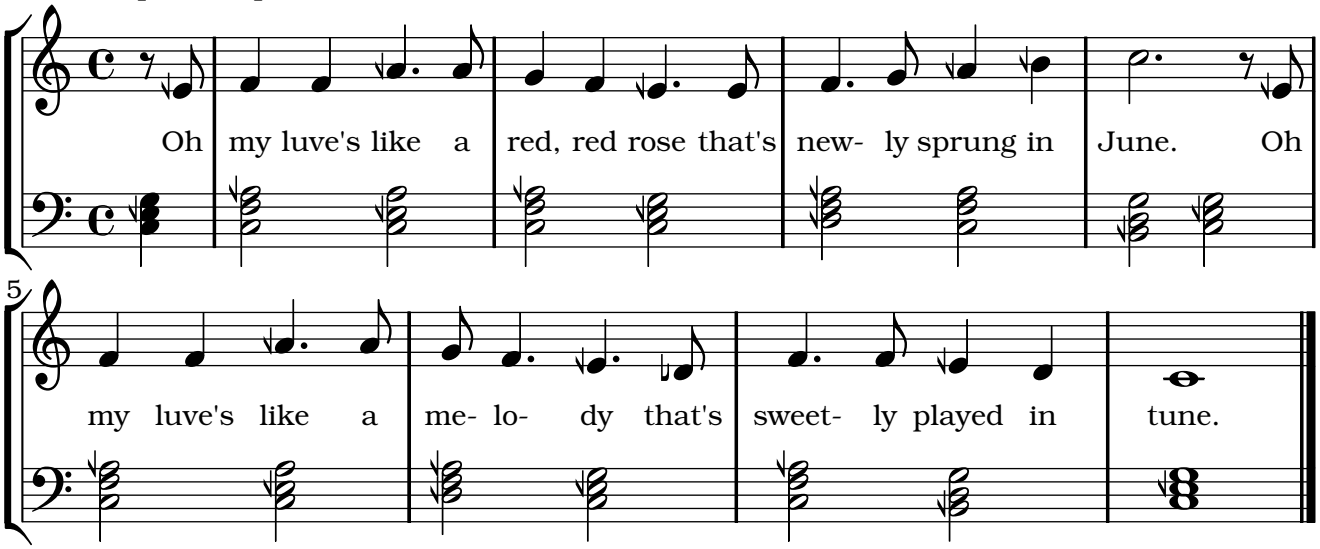
Tripod notation:



60 note equal temperament:



72 note equal temperament:



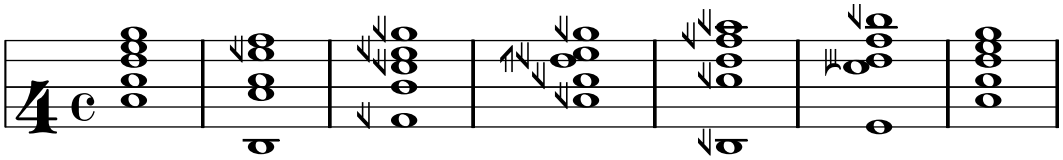
Magic Comma Pump

THE FINAL EXAMPLE is the Magic comma pump. Because it's written for Magic temperament, it looks more complicated in the Trojan notations. You can see this it below.

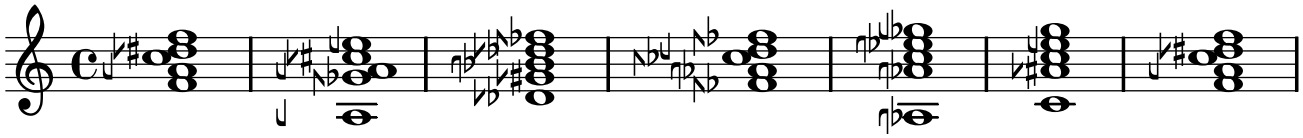
The progression only works – with a consistent tuning for Magic equivalences, and the last chord the same as the first – in Magic temperaments. 60-equal is a Magic temperament. The Tripod example uses `tripod60.ly`, and so all semitoes all the same. In 72-equal, you can see `dob` (dovabum) becoming `dof` (dovera flattened by the other semitoe) between the fourth and fifth chords. This manifests itself in a different shift being applied to the A flat in the mixed Sagittal example.

There's another note that changes its spelling in the Trojan renderings. Between the second and third chords, a tethabum (the second highest note) changes from a kind of C sharp to a kind of D flat. I chose it that way so that the third chord would be written with the note heads all in the same column. I chose it by putting `ebmf` in the score instead of `eb`. The `mf` suffix stands for "meantone flat" and says that on a what I call a meantone staff it should be written with a flat rather than a sharp. In this way, it's possible to control the output of two distinct notations.

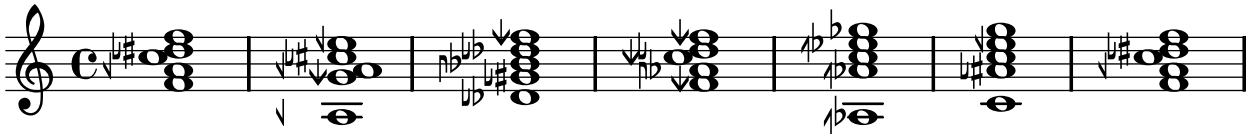
There are two other cases where I changed the default output of the Trojan notations. You can spot them by looking for the `mf` or `ms` suffixes in the source code. Maybe you work out why I made these choices, and maybe you'll disagree with me. The important thing is that we can get the same music to display different ways without being at the mercy of a dumb algorithm to convert from one pitch representation to another.



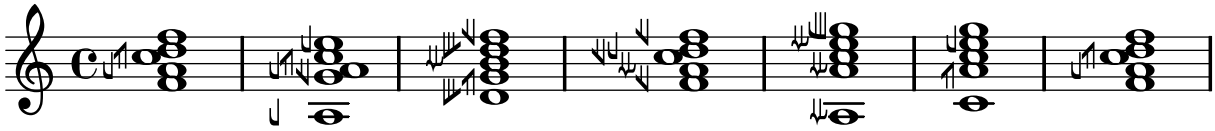
60 note equal temperament, mixed Sagittal:



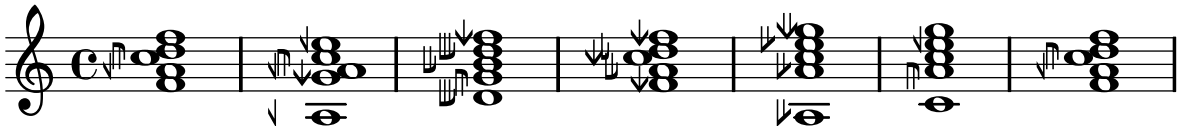
72 note equal temperament, mixed Sagittal:



60 note equal temperament, pure Sagittal:



72 note equal temperament, pure Sagittal:



LilyPond code:

```
voicea = \tripod #4 { seth1  dov, ebmf seb   dof,    tan seth }
voiceb = \tripod #4 {  dov1 leth   hov dob   dof yabudms' dov }
voicec = \tripod #5 {  tan1  dov,  yab tab   tan     tan  tan }
voiced = \tripod #5 { meth1   eb  ebmf  ej   puf     pip meth }
voicee = \tripod #5 { seth1  pip   seb seb lebmf     hof seth }
```