

Regular Mappings and Marvel Temperaments

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Goals of Regular Temperament

- New harmonic systems
- Frugal scales
- Notation
- Generalized keyboards
- Tuning

Beyond Temperament

- Support flexible pitch instruments.
- Leave the tuning as just intonation.
(Fokker 1969)
- Write just intonation as tempered notation.
(Secor 1975)

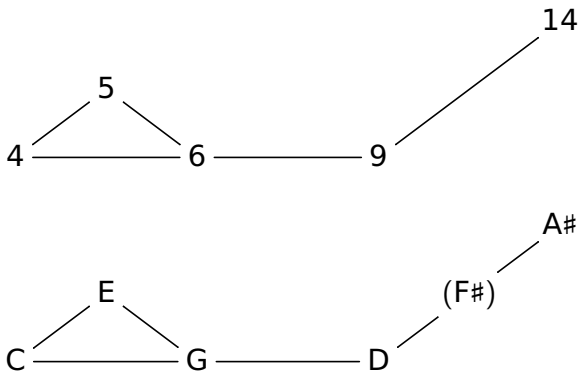
The mapping from just to tempered pitches remains.

What Took So Long?

- New pitches are disruptive.
- Meantone is good.
- Not all music needs pure harmony.
- Good mappings are easy to recognize but hard to find.
- Computers make searches and tuning easier.

The Marvel Mapping

Tempering out 225:224 (7.7 cents) relates the 7th harmonic partial to two fifths and two thirds.



Interval Arithmetic

$$\frac{5}{4} \times \frac{5}{4} \times \frac{9}{7} \approx \frac{2}{1}$$

$$\frac{5}{4} \times \frac{5}{4} \times \frac{9}{8} \approx \frac{7}{4}$$

$$\frac{5}{4} \times \frac{5}{4} \times \frac{3}{2} \times \frac{3}{2} \approx \frac{7}{2}$$

$$\frac{16}{15} \approx \frac{15}{14}$$

Question

How many semitones are there to an octave?

Answer

$$\frac{\log(2)}{\log\left(\frac{15}{14}\right)} = 10.04\dots$$

$$\frac{\log(2)}{\log\left(\frac{16}{15}\right)} = 10.74\dots$$

$$2 \frac{\log(2)}{\log\left(\frac{8}{7}\right)} = 10.38\dots$$

Scale Sizes

This week's Marvel Top 40:

19	31	12	41	22	53	10	72
50	9	34	29	7	60	43	3
84	63	65	21	2	94	91	75
32	51	28	103	48	15	40	69
81	46	113	17	79	26	16	74

Substituted Dominant Cadences

The image displays three staves of musical notation, each representing a different substituted dominant cadence in C major. Each staff begins with a treble clef and a common time signature (C). The first staff shows a sequence of three chords: C major (C-E-G), F major (F-A-C), and C major (C-E-G). The second staff shows a sequence of three chords: C major (C-E-G), F major (F-A-C), and F major (F-A-C). The third staff shows a sequence of three chords: C major (C-E-G), F major (F-A-C), and F major (F-A-C). Each chord is represented by a vertical line with a wavy pattern indicating the notes. Arrows point to the notes in each chord, and a double bar line is at the end of each staff.

Undiminishing Triads

A musical staff in treble clef with a common time signature (C). The staff contains four measures of music. The first measure has a key signature of one sharp (F#) and contains a triad with a downward arrow above it. The second measure contains a triad with a downward arrow above it. A double bar line separates the first two measures from the last two. The third measure has a key signature of one sharp (F#) and contains a triad with a downward arrow above it, labeled with a Roman numeral VI above it. The fourth measure contains a triad with a downward arrow above it, labeled with a Roman numeral I above it. The staff ends with a double bar line.

Undiminishing Triads



... we should be expecting ascending minor-third, ascending fifth, and either ascending or descending major-second progressions ...

Dmitri Tymoczko,
Doing the Time Warp from Schütz to the Beatles
<http://dmitri.tymoczko.com/>

Dominant Cadences

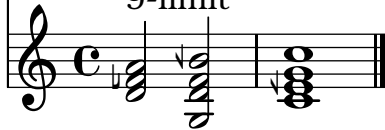
Extended 5-limit



Pythagorish



9-limit



Tritone Substitutions

The image displays three staves of musical notation, each illustrating tritone substitutions in a different temperament. The notation is in treble clef with a common time signature (C). The notes are represented by vertical stems with various symbols (dots, lines, and circles) indicating pitch and temperament. The staves are labeled as follows:

- 7-limit:** Shows a sequence of chords. The first chord is a triad (C4, E4, G4). The second chord is a dyad (F4, Bb4). The third chord is a dyad (Ab4, D5). The fourth chord is a triad (C5, Eb5, G5). The fifth chord is a triad (C5, E5, G5). The sixth chord is a triad (C5, E5, G5).
- Pythagorish:** Shows a sequence of chords. The first chord is a triad (C4, E4, G4). The second chord is a dyad (F4, Bb4). The third chord is a dyad (Ab4, D5). The fourth chord is a triad (C5, Eb5, G5). The fifth chord is a triad (C5, E5, G5). The sixth chord is a triad (C5, E5, G5).
- 9-limit:** Shows a sequence of chords. The first chord is a triad (C4, E4, G4). The second chord is a dyad (F4, Bb4). The third chord is a dyad (Ab4, D5). The fourth chord is a triad (C5, Eb5, G5). The fifth chord is a triad (C5, E5, G5). The sixth chord is a triad (C5, E5, G5).

This Week's Marvel Top 10

Meantone	19 & 31	Huygens 1661 (pub. 1724?)
Magic	19 & 41	Secor (before 1974)
Orwell	31 & 22	2001?
Miracle	31 & 41	Secor 1975
Garibaldi	12 & 41	Bosanquet 1875
Pajara	12 & 22	Erlich 1998
Catakleismic	19 & 53	Secor (before 1974)
Negri	19 & 10	Negri 1986
Waage	12 & 72	Waage 1985?
Würschmidt	31 & 34d	Würschmidt 1921?

Meantone

- The 7-limit optimum is close to quarter-comma.
- A 7:4 is an augmented sixth.
- 7-limit unique.
- Two 4:5:6:7 tetrads in a 12 note scale.
- e.g. $E\flat-G-B\flat-C\sharp$.

Magic

- Major third generator.
- Optimal tuning close to 41-equal.
- Seven 4:5:6:7 tetrads in a 19 note scale.
- 9-limit unique.
- Every 9-limit interval better than 12-equal.
- Lacking nice diatonics.

Magic Arithmetic

$$\frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \approx \frac{3}{1}$$

$$\frac{9}{7} \times \frac{9}{7} \approx \frac{5}{3}$$

$$\frac{36}{35} \approx \frac{25}{24}$$

$$\frac{245}{243} \approx \frac{1}{1}$$

Orwell

- Optimal 19/84 generator.
- Two 4:5:6:7 tetrads in a 13 note scale.
- 9-limit unique.
- Simple 11-limit extension.
- 9 note scale with dense harmony.

Orwell Arithmetic

$$\frac{7}{6} \times \frac{7}{6} \times \frac{7}{6} \approx \frac{8}{5}$$

$$\frac{6}{5} \times \frac{8}{7} \times \frac{9}{7} \approx \frac{7}{4}$$

$$\frac{7}{6} \times \frac{7}{6} \approx \frac{11}{8}$$

$$\frac{8}{7} \times \frac{12}{11} \approx \frac{5}{4}$$

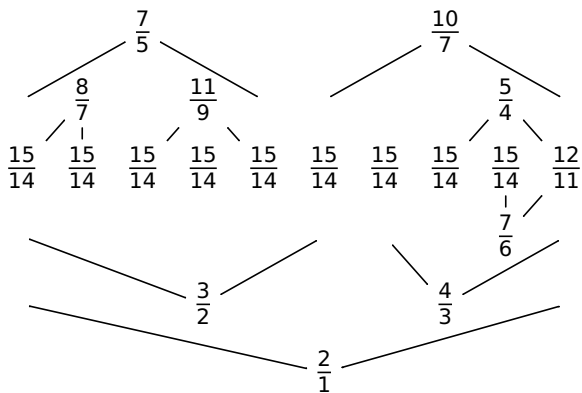
Minerva

Marvel

Miracle

- Semitone generator.
- Optimal tuning close to 72-equal.
- Eight 4:5:6:7 tetrads in a 21 note scale.
- 11-limit unique.
- 10 note near-equal scale.

Decimal Scale



Garibaldi (Schismatic)

- Near-Pythagorean tuning.
- Two 4:5:6:7 tetrads in a 17 note scale.
- 9-limit unique.
- Positive keyboard mapping.
(Bosanquet 1875, Wilson 1975)
- Elusive Arab/Persian connection.

Thank You

For references, slides and examples, see:

<http://x31eq.com/Aberdeen>