# Regular Mappings and Marvel Temperaments

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# Goals of Regular Temperament

- New harmonic systems
- Frugal scales
- Notation
- Generalized keyboards
- Tuning

# Beyond Temperament

- Support flexible pitch instruments.
- Leave the tuning as just intonation.
  (Fokker 1969)
- Write just intonation as tempered notation.
  (Secor 1975)

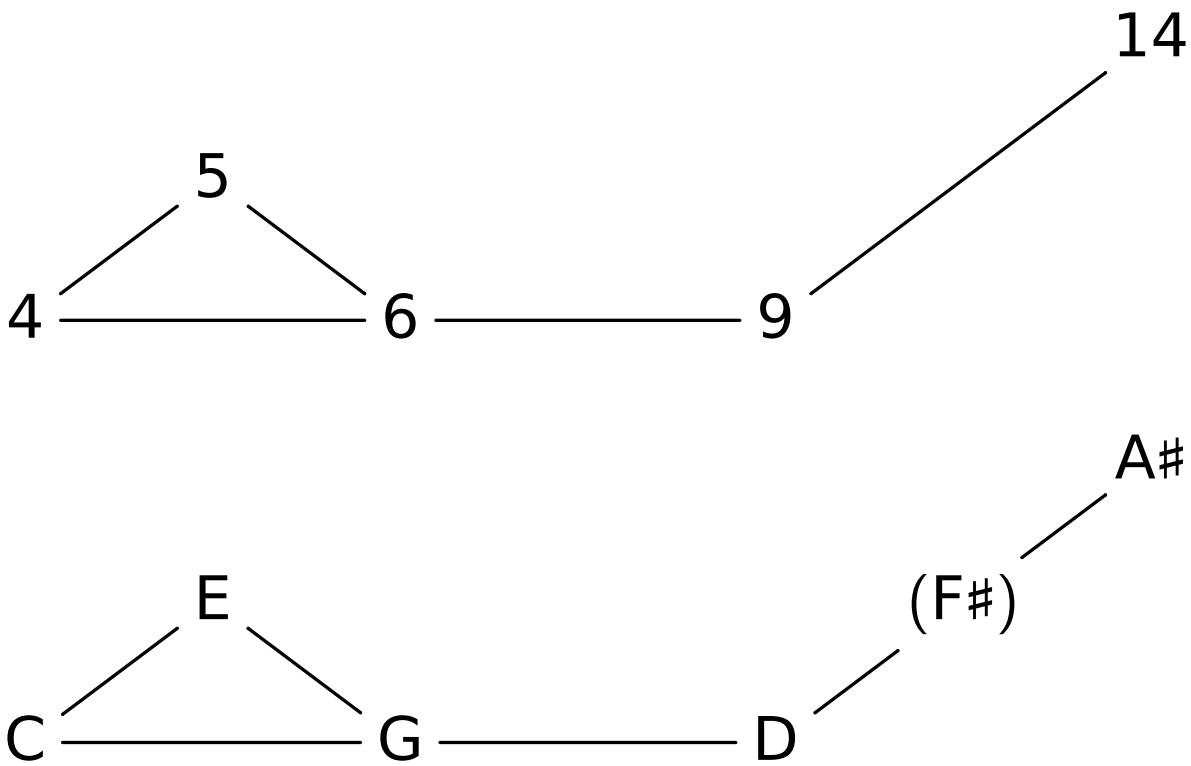
The mapping from just to tempered pitches remains.

# What Took So Long?

- New pitches are disruptive.
- Meantone is good.
- Not all music needs pure harmony.
- Good mappings are easy to recognize but hard to find.
- Computers make searches and tuning easier.

# The Marvel Mapping

Tempering out 225:224 (7.7 cents) relates the 7th harmonic partial to two fifths and two thirds.



# Interval Arithmetic

$$\frac{5}{4} \times \frac{5}{4} \times \frac{9}{7} \approx \frac{2}{1}$$

$$\frac{5}{4} \times \frac{5}{4} \times \frac{9}{8} \approx \frac{7}{4}$$

$$\frac{5}{4} \times \frac{5}{4} \times \frac{3}{2} \times \frac{3}{2} \approx \frac{7}{2}$$

$$\frac{16}{15} \approx \frac{15}{14}$$

# Question

How may semitones are there to an octave?

# Answer

$$\frac{\log(2)}{\log\left(\frac{15}{14}\right)} = 10.04\ldots$$

$$\frac{\log(2)}{\log\left(\frac{16}{15}\right)} = 10.74\ldots$$

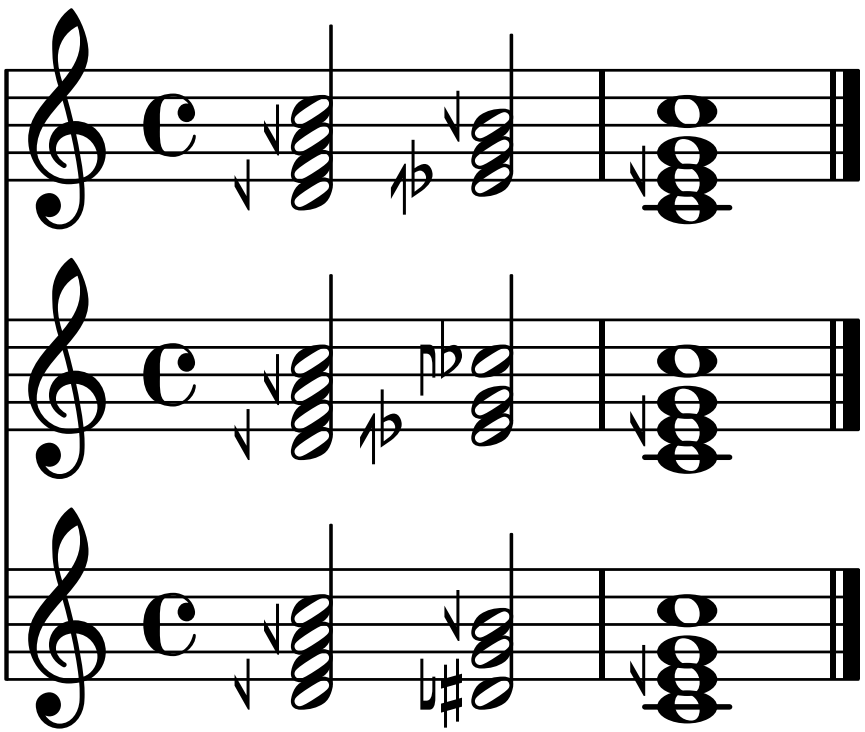
$$2\frac{\log(2)}{\log\left(\frac{8}{7}\right)} = 10.38\ldots$$

# Scale Sizes

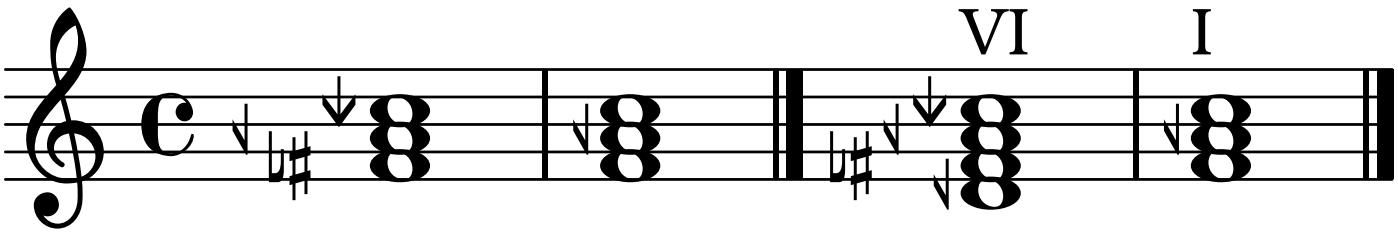
This week's Marvel Top 40:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 19 | 31 | 12 | 41 | 22 | 53 | 10 | 72 |
| 50 | 9 | 34 | 29 | 7 | 60 | 43 | 3 |
| 84 | 63 | 65 | 21 | 2 | 94 | 91 | 75 |
| 32 | 51 | 28 | 103 | 48 | 15 | 40 | 69 |
| 81 | 46 | 113 | 17 | 79 | 26 | 16 | 74 |

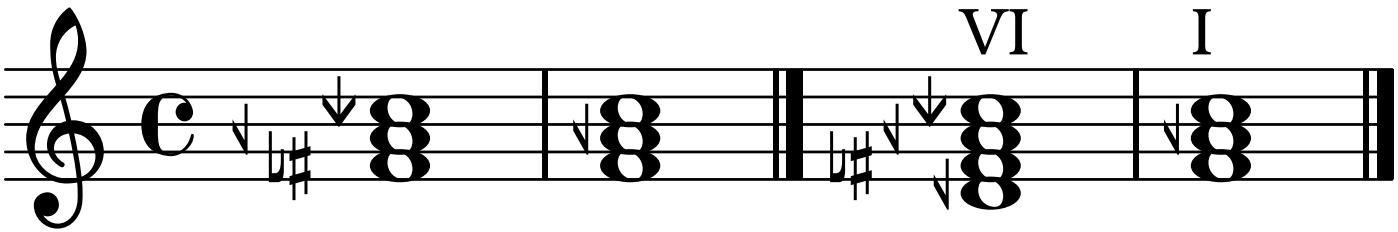
# Substituted Dominant Cadences



# Undiminishing Triads



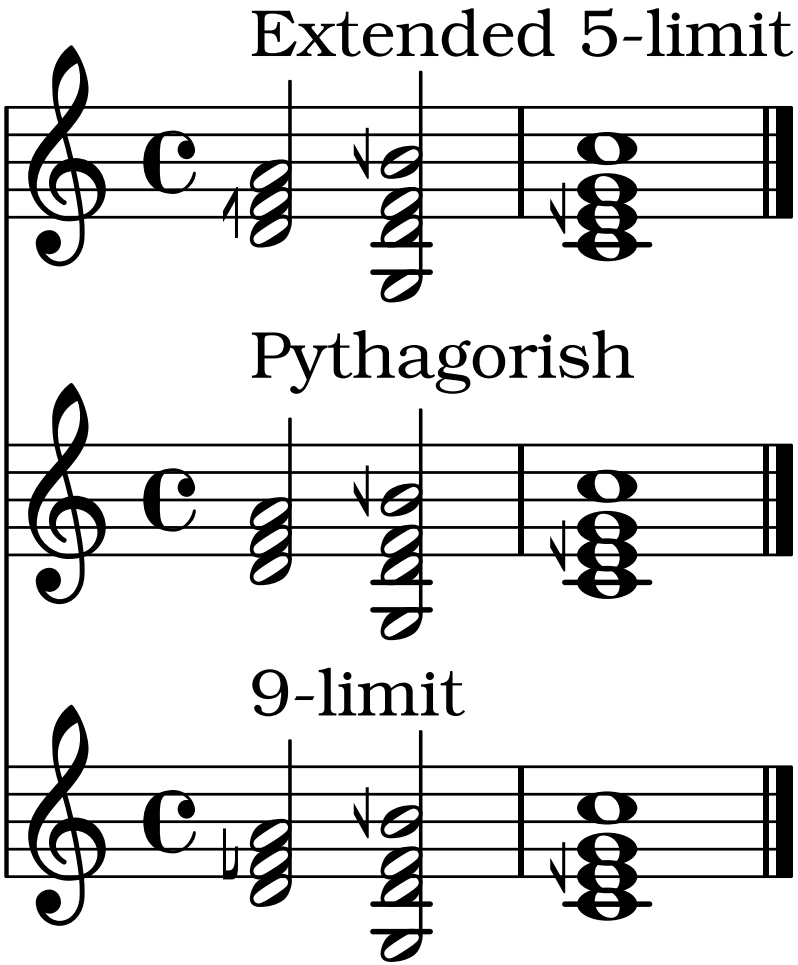
# Undiminishing Triads



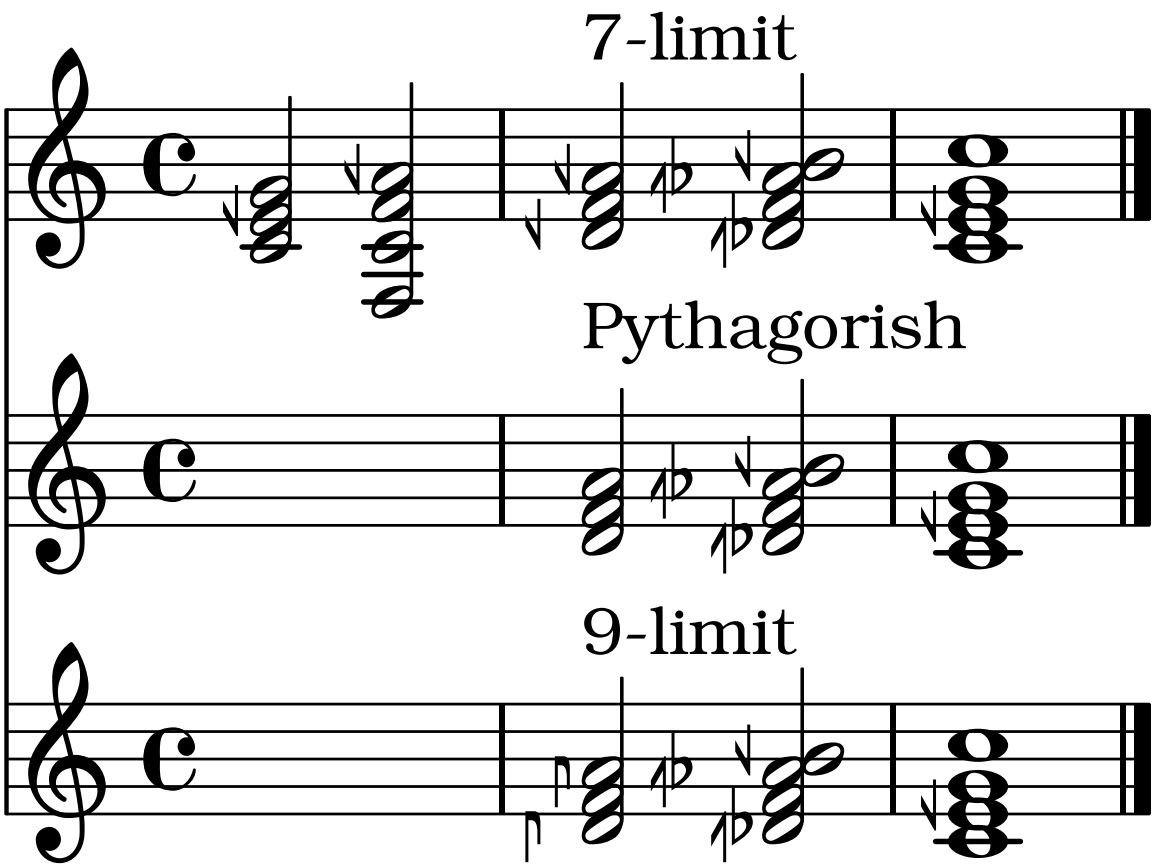

> *. . . we should be expecting ascending minor-third, ascending fifth, and either ascending or descending major-second progressions . . .*

Dmitri Tymoczko,
*Doing the Time Warp from Schütz to the Beatles*
`http://dmitri.tymoczko.com/`

# Dominant Cadences



# Tritone Substitutions



# This Week's Marvel Top 10

| | | |
|---|---|---|
| Meantone | 19 & 31 | Huygens 1661 (pub. 1724?) |
| Magic | 19 & 41 | Secor (before 1974) |
| Orwell | 31 & 22 | 2001? |
| Miracle | 31 & 41 | Secor 1975 |
| Garibaldi | 12 & 41 | Bosanquet 1875 |
| Pajara | 12 & 22 | Erlich 1998 |
| Catakleismic | 19 & 53 | Secor (before 1974) |
| Negri | 19 & 10 | Negri 1986 |
| Waage | 12 & 72 | Waage 1985? |
| Würschmidt | 31 & 34d | Würschmidt 1921? |

# Meantone

- The 7-limit optimum is close to quarter-comma.
- A 7:4 is an augmented sixth.
- 7-limit unique.
- Two 4:5:6:7 tetrads in a 12 note scale.
- e.g. E♭–G–B♭–C♯.

# Magic

- Major third generator.
- Optimal tuning close to 41-equal.
- Seven 4:5:6:7 tetrads in a 19 note scale.
- 9-limit unique.
- Every 9-limit interval better than 12-equal.
- Lacking nice diatonics.

# Magic Arithmetic

$$\frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \approx \frac{3}{1}$$

$$\frac{9}{7} \times \frac{9}{7} \approx \frac{5}{3}$$

$$\frac{36}{35} \approx \frac{25}{24}$$

$$\frac{245}{243} \approx \frac{1}{1}$$

# Orwell

- Optimal 19/84 generator.
- Two 4:5:6:7 tetrads in a 13 note scale.
- 9-limit unique.
- Simple 11-limit extension.
- 9 note scale with dense harmony.

# Orwell Arithmetic

$$\frac{7}{6} \times \frac{7}{6} \times \frac{7}{6} \approx \frac{8}{5}$$

$$\frac{6}{5} \times \frac{8}{7} \times \frac{9}{7} \approx \frac{7}{4}$$

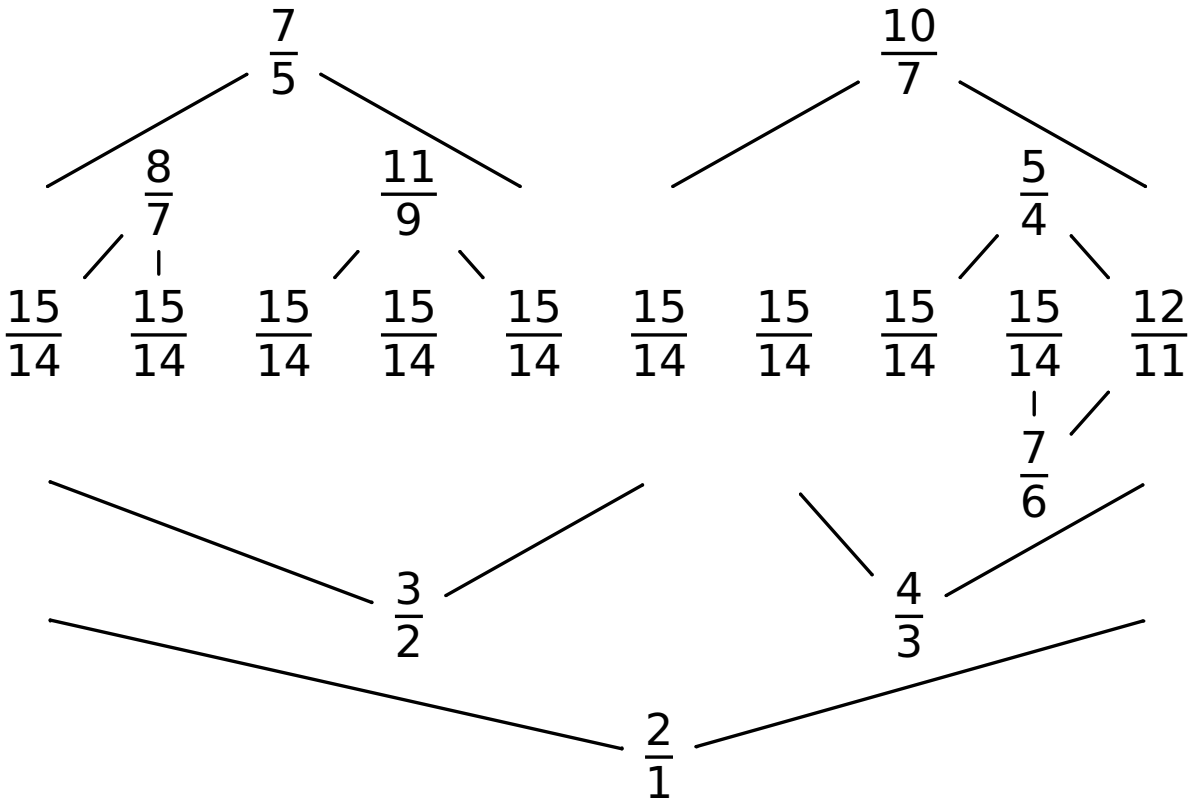
$$\frac{7}{6} \times \frac{7}{6} \approx \frac{11}{8}$$ Minerva

$$\frac{8}{7} \times \frac{12}{11} \approx \frac{5}{4}$$ Marvel

# Miracle

- Semitone generator.
- Optimal tuning close to 72-equal.
- Eight 4:5:6:7 tetrads in a 21 note scale.
- 11-limit unique.
- 10 note near-equal scale.

# Decimal Scale



# Garibaldi (Schismatic)

- Near-Pythagorean tuning.
- Two 4:5:6:7 tetrads in a 17 note scale.
- 9-limit unique.
- Positive keyboard mapping.
  (Bosanquet 1875, Wilson 1975)
- Elusive Arab/Persian connection.

# Thank You

For references, slides and examples, see:

`http://x31eq.com/Aberdeen`